The Macroeconomics of Imperfect Capital Markets

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Lecture 10: Equilibrium Selection in Models of Multiple Equilibria

Morris and Shin (1998)

Morris and Shin (AER, 1998): Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks

Problems of multiple equilibrium models:

- Equilibrium selection (e.g. onset of crises) not pinned down by theory
- Policy questions relating to vulnerability to crisis cannot be tackled

Global Games Approach to Resolving Multiplicity:

- Carlsson and van Damme (Ecta, 1993) introduces concept of "global games:" incomplete information games where (small) uncertainty about payoff structure induces players to select the risk-dominant equilibrium (similar to Selten's "trembling hand" equilibria)
- Morris and Shin apply this to speculative attacks to determine the unique equilibrium in such models

Main Insights

Main insights:

- Information structure is crucial to decision to attack (run):
  - optimal to attack when others also attack
  - optimal to refrain from attacking when others refrain

  $\longrightarrow$ strategic complementarities in decision making

- Assume each investor receives a noisy signal about fundamentals
  $\longrightarrow$ common knowledge no longer holds
  $\longrightarrow$ investors need to form beliefs about signals of others
  $\longrightarrow$ sufficient to coordinate to a unique equilibrium

Simple Model of Speculative Attacks

Structure of the economy:

- State of fundamentals is captured by $\theta \sim U[0,1]$ (higher θ equals stronger fundamentals)
- "Shadow" exchange rate $f(\theta)$ if there was no intervention satisfies $f'(\theta) > 0$
- government has pegged exchange rate to $e^* \geq f(\theta)\ \forall\ \theta$

Two sets of actors:

- Unit mass of speculators in foreign exchange market: decide whether to attack by selling 1 unit or not
  - cost of attacking is t
  - payoff if successful is $e^* - f(\theta) - t$

  denote total mass of speculators who attack α
- Government: chooses whether to defend peg
  - benefit of defending is v
  - cost of defending is $c(\alpha, \theta)$ where $c_\alpha > 0 > c_\theta$

Simple Model of Speculative Attacks

Assumptions about functional forms:

- $c(0,0) > v$: in worst state, it's not worth defending the peg even if noone attacks
- $c(1,1) > v$: in best state, it's not worth defending the peg if everyone attacks
- $e^* - f(1) < t$: in best state, speculators cannot recoup transaction costs

Threshold values:

- define $\underline{\theta}$ such that $c(0,\underline{\theta}) = v$: lower bound of states in which government defends peg
- define $\bar{\theta}$ such that $f(\bar{\theta}) = e^* - t$: upper bound of states in which investors attack

Thresholds values

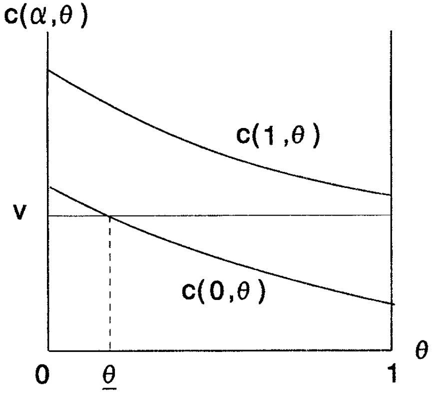

FIGURE 1. COST AND BENEFIT TO THE GOVERNMENT IN MAINTAINING THE CURRENCY PEG

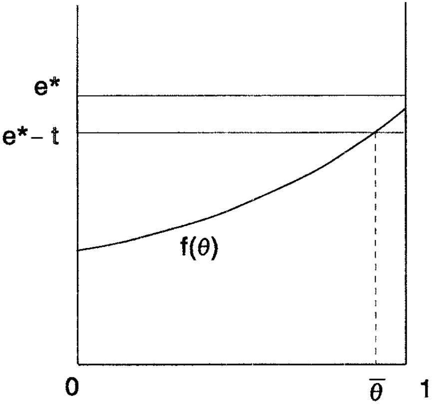

FIGURE 2. THE MANAGED EXCHANGE RATE AND THE EXCHANGE RATE IN THE ABSENCE OF INTERVENTION AS A FUNCTION OF THE STATE OF FUNDAMENTALS

Possible Equilibria

Tripartite classification of fundamentals:

- for $\theta \in [0, \underline{\theta}]$: currency unstable
  $\longrightarrow$ attack always occurs
- for $\theta \in (\underline{\theta}, \bar{\theta})$: currency ripe for attack
  $\longrightarrow$ multiple equilibria
  $\longrightarrow$ interesting region
- for $\theta \in [\bar{\theta}, 1]$: currency stable $\longrightarrow$ no speculative attacks

Game with Imperfect Information

Setup of Imperfect Information:

- nature chooses θ
- speculators observe i.i.d. signals $x \sim U[\theta - \varepsilon, \theta + \varepsilon]$ for small ε and decide whether to attack
- government observes θ and fraction α of attackers and decides whether to defend the peg

Equilibrium:

- strategies for speculators and government such that no agent has an incentive to deviate
- define $a(\theta)$ as the mass of attackers such that the government is indifferent between defending or not, i.e. $c(a(\theta), \theta) = v$
  $\rightarrow$ defend iff $\alpha \leq a(\theta)$

Simple Model of Speculative Attacks

Thresholds value for government

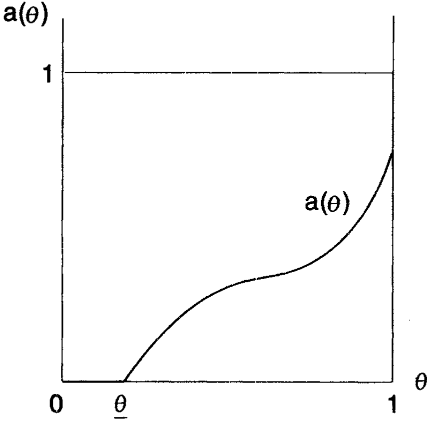

FIGURE 3. THE PROPORTION OF SPECULATORS WHOSE SHORT SALES ARE SUFFICIENT TO INDUCE DEPRECIATION, EXPRESSED AS A FUNCTION OF THE FUNDAMENTALS

Stratgies of Speculators

For a given strategy π of speculators:

- denote selling decision for a signal x as $\pi(x) \in \{0, 1\}$
- denote mass of speculators selling, given fundamental θ as

$$s(\theta; \pi) = \frac{1}{2\varepsilon} \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x)\, dx$$

- denote the set of θ's where government abandons peg as

$$A(\pi) = \{\theta : s(\theta; \pi) \geq a(\theta)\}$$

- reduced-form payoff of speculators (given θ)

$$h(\theta; \pi) = \begin{cases} e^* - f(\theta) - t & \text{if } \theta \in A(\pi) \\ -t & \text{if } \theta \notin A(\pi) \end{cases}$$

- reduced-form payoff of attacking (given signal x)

$$u(x; \pi) = \frac{1}{2\varepsilon} \int_{x-\varepsilon}^{x+\varepsilon} h(\theta; \pi)\, d\theta = \frac{1}{2\varepsilon} \int_{A(\pi) \cap [x-\varepsilon, x+\varepsilon]} [e^* - f(\theta)]\, d\theta - t$$

Equilibrium Determination

Lemma (Strategic Complementarities)

If $\pi(x) \geq \pi'(x) \, \forall x$ then $u(x;\pi) \geq u(x;\pi')$

Define threshold strategy $I_k = 1_{\{x<k\}}$ of attacking when signal $x < k$
$u(k, I_k)$ is payoff of marginal investor under strategy I_k if signal is k

Lemma (Threshold strategy)

The function $u(k, I_k)$ is continuous and strictly decreasing in k

Lemma (Uniqueness)

There is a unique x^* such that speculators attack iff $x < x^*$, which is obtained by solving for $u(k, I_k) = 0$

Equilibrium Determination

Aggregate mass of short-sales:

$$s\left(\theta; I_{x^*}\right) = \begin{cases} 1 & \text{if } \theta < x^* - \varepsilon \\ \frac{1}{2} - \frac{1}{2\varepsilon}\left(\theta - x^*\right) & \text{if } \theta \in \left[x^* - \varepsilon, x^* + \varepsilon\right] \\ 0 & \text{if } \theta > x^* + \varepsilon \end{cases}$$

- $s\left(\theta; I_{x^*}\right)$ is decreasing in θ
- $a\left(\theta\right)$ is increasing in θ

Theorem (Uniqueness of Equilibrium)

There is a unique θ^* such that the peg is abandoned iff $\theta \leq \theta^*$

Theorem (Limiting Case)

In the limit as $\varepsilon \to 0$, the threshold θ^* is given by $f\left(\theta^*\right) = e^* - 2t$

(Note: expected payoff for marginal investor $\frac{1}{2}\left[e^* - f\left(\theta^*\right)\right]$, expected cost t)

Derivation of Optimal Strategy θ^*

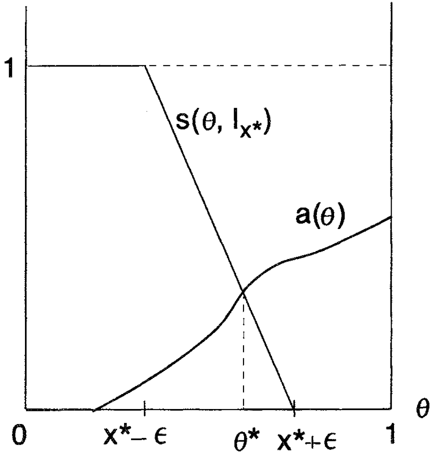

FIGURE 4. THE DERIVATION OF THE CUTOFF POINT FOR THE STATE OF FUNDAMENTALS AT WHICH THE EQUILIBRIUM SHORT SALES ARE EQUAL TO THE SHORT SALES WHICH INDUCE DEPRECIATION

Comparative Statics

- Changes in transaction costs:
  higher transaction costs reduce incidence of crises

- Changes in aggregate wealth of speculators (hot money):
  higher wealth per investor reduces $a(\theta)$ and raises incidence of crises (with the magnitude of the effect depending on ε)