The Macroeconomics of Imperfect Capital Markets

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Lecture 11: Bubbles and Crashes

Bubbles Throughout History

Tulip Mania (1620 – 1637):

- introduced from Ottoman Empire in 16th century
- important status symbol
- top price for Semper Augustus: 40fold average yearly income
- speculation using high leverage
- crashed in Feb. 1637
- followed by economy-wide depression

Bubbles Throughout History

South Sea Bubble (1711 – 1720):

- monopoly on trade/slave trade with America
- took on large amount of British government debt (debt/equity swap)
- accompanied by a frenzy of other IPOs:

  "A company for carrying on an undertaking of great advantage,

  but nobody to know what it is."
- at the same time in France: Mississippi Bubble (1720)

Railway Mania of 1840s, popped in 1846

Roaring '20s, preceding market crash of 1929:

- five-fold increase of DJIA between 1924 and 1929
- burst on Black Thursday, October 24, 1929
- followed by Black Monday and Black Tuesday
- market bottomed in 1932
- level of 1929 was not re-gained until 1954

Economic Models of Bubbles

Definition of bubble:
Price of an asset exceeding the asset's fundamental value (e.g. PDV), typically because holders expect that the price will rise further in future

Categories of bubbles (Brunnermeier, New Palgrave):

1. Rational expectations and symmetric information
2. Asymmetric information
3. Behavioral traders and limits to arbitrage
4. Heterogeneous beliefs

Rational Bubbles

Rational expectations equilibrium with full information:
price of asset with dividends $\{d_{t+s}\}_{s=0}^{\infty}$ and required rate of return ρ

$$p_t = E_t\left[\frac{p_{t+1} + d_{t+1}}{1+\rho}\right]$$

Solution in infinite horizon world:

$$p_t = v_t + b_t \quad \text{where} \quad \underbrace{v_t = E_t\left[\sum\left(\frac{1}{1+\rho}\right)^s d_{t+s}\right]}_{\text{fundamental value}} \quad \text{and} \quad \underbrace{b_t = \frac{E\left[b_{t+1}\right]}{1+\rho}}_{\text{bubble component}}$$

To obtain uniqueness: impose **transversality condition**
$\lim_{s\to\infty} E_t\left[\frac{p_{t+s}}{(1+\rho)^s}\right] = 0$
this rules out bubbles, i.e. it pins down $b_t = 0 \quad \forall t$

Existence of Rational Bubbles

Santos and Woodford (Econometrica, 1997),
Rational Asset Pricing Bubbles: rationality rules out most bubbles

- with *finite* horizon bubbles cannot exist: $p_T = d_T$ pins down $b_T = 0$
- bubbles cannot emerge from nowhere
  $\rightarrow$ must have existed in all earlier periods
- must be expected to continue forever with non-zero probability
- if there exists an upper limit on the size of a bubble, it cannot exist by backward induction
  - examples: pyramid schemes, commodities with close substitutes
  - if $\rho > \dot{Y}$ bubble would outgrow world wealth
  - negative bubble $b_t < 0$ cannot exist with limited liability because $p_t \geq 0$
- bubbles only rational if aggregate wealth is infinite-valued
  because economy is not sufficiently productive (low interest rate)

Rational Bubbles

Rational bubble term in Blanchard and Watson (1982):
General property required: $E_t [b_{t+1}] = (1+\rho) b_t$
Example:

$$b_{t+1} = \begin{cases} \frac{(1+\rho) b_t}{\pi} & \text{with probability } \pi \\ 0 & \text{with probability } 1-\pi \end{cases}$$

$\rightarrow$ bubbles grow at faster rate than required return (until bursting)

The Limits of Arbitrage

Shleifer and Vishny (1997): The Limits of Arbitrage
Arbitrage can deflate any bubble, but there are limits to arbitrage

Textbook definition of arbitrage:

- risk-less market transaction
- requires no capital
- yields positive profit with probability one

→ fundamental concept in finance
→ much of finance theory is built on "no arbitrage" condition

Real world: 'risk arbitrage:'

- arbitrageurs need to put up capital (e.g. margin requirement)
- they may have to inject more capital → danger of illiquidity
- arbitrage usually performed by specialists using other people's money (e.g. hedge funds)
- profit does not result in all states of the world

The Limits of Arbitrage

An Agency Model of Limited Arbitrage:

- three time periods: $t = 1, 2, 3$
- three sets of agents: competitive arbitrageurs, noise traders, investors
- asset with fundamental value V at $t = 3$, known only to arbitrageur

Behavior of (irrational) noise traders:

- at $t = 1$, a random 'pessimism' shock denoted $S_1 > 0$ occurs
- at $t = 2$, misperception increases $S_2 > S_1$ with probability q
- noise traders have a demand for assets $Q^N = \left[V - S_t\right] / p_t$

Description of arbitrageurs:

- risk-neutral, access to costless storage technology
- amount of resources under management F_t (F_1 given)
- at $t = 2$, invest all of F_2 if $S_1 \neq V$ to make profit with certainty: asset demand $Q_2^A = F_2 / p_2 \;\; \rightarrow \;\; p_2 = V - S_2 + F_2$ if $F_2 < S_2$
- at $t = 1$, invest $D_1 \leq F_1$ in risky asset for arbitrage

The Limits of Arbitrage

Behavior of investors:

- at $t = 1$, provide F_1 in resources to arbitrageurs
- at $t = 2$, infer skill of arbitrageurs from past returns

$$R = \frac{D_1}{F_1} \cdot \frac{p_2}{p_1} + \frac{F_1 - D_1}{F_1}$$

- adjust funds provided to arbitrageurs to $F_2 = F_1 \cdot G(R)$ where $G(1) = 1$ and $G' \geq 1$, e.g. $G(R) = aR + (1 - a)$
  for $a < 1$ investors inject funds after bad performance
  for $a = 1$ investors are passive
  for $a > 1$ investors withdraw funds after bad performance

$\rightarrow$ in latter case, if asset price moved away from fundamentals, arbitrageurs have less funds available for arbitrage
$\rightarrow$ fundamental problem of performance-based arbitrage

Optimization Problem of Arbitrageurs

Optimization problem of arbitrageurs:

$$\begin{aligned}\max EW &= (1-q)\overbrace{\left\{a\left(\frac{D_1\cdot V}{p_1}+F_1-D_1\right)+(1-a)\,F_1\right\}}^{\text{profit if asset price returns to fundamental}} \\ &\quad +q\cdot\frac{V}{p_2}\underbrace{\left\{a\left(\frac{D_1\cdot p_2}{p_1}+F_1-D_1\right)+(1-a)\,F_1\right\}}_{\text{profit if asset price diverges from fundamental}}\end{aligned}$$

First-order condition for raising D_1:

$$\underbrace{(1-q)\left(\frac{V}{p_1}-1\right)}_{\text{benefit from arbitrage at } t=1} + \underbrace{q\left(\frac{p_2}{p_1}-1\right)\frac{V}{p_2}}_{\text{loss from not having funds at } t=2} \geq 0$$

Invest more at time 1 if:

- probability q of further decline is low
- deviation from fundamental $V-S_1$ is high
- potential loss S_2-S_1 is small,

Amplification Under Performance-Based-Arbitrage

Performance-based arbitrage ($a > 0$) makes markets less efficient: funds available for arbitrage decline when they are most needed

Potential for Amplification:

- if $a > 1$, then investors actively withdraw funds after bad performance
- if arbitrageurs have too little liquidity available (D_1 close to F_1), they have to sell assets to obtain liquidity
- this pushes down the market price further than without arbitrageurs
- the greater a and the smaller $F_1 - D_1$, the stronger amplification
- in operating companies equity cushions liquidity problem of debt in arbitrage funds, all of equity can be withdrawn

Economic Growth with Bubbles

Martin and Ventura (AER, 2012): Economic Growth with Bubbles

Bubbles are alternative stores of value:

- capital: costly to produce, but useful in production
- bubbles: free to initiate, but useless in production
  - $\longrightarrow$ builds on Samuelson (1958), Tirole (1985)
  - $\longrightarrow$ macroeconomic pyramid scheme to remedy dynamic inefficiency
  - $\longrightarrow$ bubbles raise consumption by reducing inefficient investment and output = *counterfactual*
  - $\longrightarrow$ bubbles are deterministic and never burst

Contribution:

- bubbles are stochastic (investor sentiment shocks)
- financial frictions: efficient and inefficient investments coexist
- bubbles reallocate capital from inefficient to efficient investments and typically raise output

Economic Growth with Bubbles

Bubbles as Pyramid Schemes:

- Assume OLG setup:
  - young households work and invest (high/low productivity)
  - old households consume

- Focus on market for bubbles:
  - supply of bubbles: old households + new bubble creators
    $\longrightarrow$ use revenue for productive investments
  - demand for bubbles: new unproductive households:
    $\longrightarrow$ bubbles = better return than unproductive investments

- In practice: bubbles attached to real assets
  (e.g. stocks, real estate, ...)

Economic Growth with Bubbles

Model Setup:

- OLG setup: each generation lives two periods (young, old)
- individuals value old-age consumption $U = E\left[c_{it+1}\right]$
- young supply one unit of labor $l_t = 1$

- Cobb-Douglas production: $F = A_{it} k_t^{\alpha} l_t^{1-\alpha}$
  $\rightarrow w_t = (1-\alpha) A_{it} k_t^{\alpha}$, $r_t = \alpha A_{it} k_t^{\alpha-1}$
- fraction ε of young is productive: $A_{it} = 1$, others $A_{it} = \delta < 1$
  $\rightarrow$ set of productive P_t and unproductive U_t investors
- financial friction prevents borrowing/lending

- young save labor income (fraction $s = (1-\alpha)$ of output)
- aggregate productivity $A = \varepsilon + (1-\varepsilon)\delta$
- equilibrium without bubbles: $k_{t+1} = sAk_t^{\alpha}$

Introduction of Bubbles:

- b_t ... market price of all exisiting bubbles
- b_t^P, b_t^U ... new bubbles by P and U
- $\left\{b_t, b_t^P, b_t^U\right\}_{t=0}^{\infty}$ ... stochastic process for bubbles
- $h_t = \left\{b_t, b_t^P, b_t^U\right\}$ ... realization in period t
  $h^t = \left\{h_0, ..., h_t\right\}$ ... history of realizations
  H_t ... set of possible histories

Definition

A stochastic process $\left\{b_t, b_t^P, b_t^U\right\}_{t=0}^{\infty}$ is an equilibrium bubble if (i) $b_t + b_t^P + b_t^U > 0$ in some t and $h^t \in H_t$ and (ii) there exists a sequence $\left\{k_t\left(h^t\right)\right\}_{t=0}^{\infty}$ that satisfies individual maximization and market clearing for all t and $h^t \in H_t$.

Return on bubbles: $R_t = \frac{b_{t+1}}{b_t + b_t^P + b_t^U}$

Economic Growth with Bubbles

Equilibrium with Bubbles: 2 Equations

1. If $b_t + b_t^P + b_t^U > 0$, bubble must deliver adequate return (note: $(1-\varepsilon)\, sk_t^\alpha$ is demand for saving of U)
   - if $b_t + b_t^P < (1-\varepsilon)\, sk_t^\alpha$ then $E_t[R_t] = \delta\alpha k_{t+1}^{\alpha-1}$
   - if $b_t + b_t^P = (1-\varepsilon)\, sk_t^\alpha$ then $E_t[R_t] \in \left[\delta\alpha k_{t+1}^{\alpha-1}, \alpha k_{t+1}^{\alpha-1}\right]$
   - if $b_t + b_t^P > (1-\varepsilon)\, sk_t^\alpha$ then $E_t[R_t] = \alpha k_{t+1}^{\alpha-1}$
2. Bubble can't exceed aggregate savings: $0 \leq b_t \leq sk_t^\alpha$

Dynamics of Capital Stock:

- if $b_t + b_t^P < (1-\varepsilon)\, sk_t^\alpha$: marginal buyer of bubble unproductive
  then $k_{t+1} = sAk_t^\alpha + (1-\delta)\, b_t^P - \delta b_t$
  $\rightarrow$ crowding out effect of bubbles b_t
  $\rightarrow$ reallocation effect of bubbles b_t^P
- if $b_t + b_t^P \geq (1-\varepsilon)\, sk_t^\alpha$: marginal buyer of bubble productive
  then $k_{t+1} = sk_t^\alpha - b_t$

Recursive Notation of 2 Equlibrium Equations:

Denote $x_t = \frac{b_t}{sk_t^\alpha}$ and same for x_t^P and x_t^U

1. If $x_t + x_t^P + x_t^U > 0$, bubble must deliver adequate return:
   - if $x_t + x_t^P < 1 - \varepsilon$ then $E_t x_{t+1} = \frac{\alpha}{s} \cdot \frac{\delta\left(x_t + x_t^P + x_t^U\right)}{A + (1-\delta)x_t^P - \delta x_t}$
   - if $x_t + x_t^P = 1 - \varepsilon$ then $E_t x_{t+1} \in \left[\frac{\alpha}{s} \cdot \frac{\delta\left(x_t + x_t^P + x_t^U\right)}{A + (1-\delta)x_t^P - \delta x_t}, \frac{\alpha}{s} \cdot \frac{x_t + x_t^P + x_t^U}{1 - x_t}\right]$
   - if $x_t + x_t^P > 1 - \varepsilon$ then $E_t x_{t+1} = \frac{\alpha}{s} \cdot \frac{x_t + x_t^P + x_t^U}{1 - x_t}$
2. Bubble can't exceed aggregate savings: $0 \leq x_t \leq 1$

Sources of shocks:

- new bubble creation x_t^P, x_t^U
- old bubble valuation: x_t

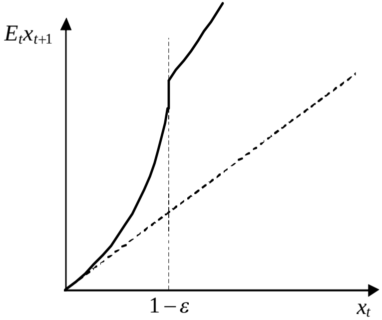
Economic Growth with Bubbles

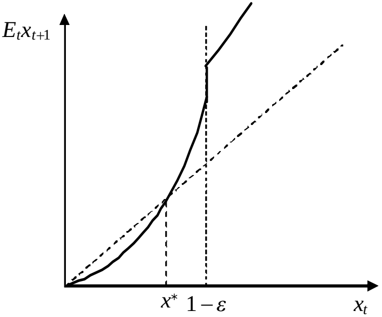
Existence of Bubbles: bubbly episodes are possible if

$$\alpha < \begin{cases} s \cdot \frac{A}{\delta} & \text{if } A > 1-\varepsilon \\ s \cdot \frac{A}{\delta} \cdot \max\left\{1, \frac{1}{4(1-\varepsilon)A}\right\} & \text{if } A \leq 1-\varepsilon \end{cases}$$

- consider first a single episode of bubble creation $x_{t0}^P + x_{t0}^U > 0$:
  - if $\alpha > s\frac{A}{\delta}$, then $E_t x_{t+1} > x_t$ at all times
    $\rightarrow$ bubble needs to grow continuously
  - if $\alpha \leq s\frac{A}{\delta}$, then interior solution possible
    $\rightarrow$ rational bubble can exist
- subsequent bubble creation by U keeps result unchanged (old bubble and new bubble compete)
- subsequent bubble creation by P is different:
  - it shifts $E_t x_{t+1}$ schedule upwards if $x_t \in (0, A] \cup (1-\varepsilon, 1]$,
  - downwards otherwise
  - intuitively, it raises investment and wage income next period
    $\rightarrow$ may make bubbles more affordable

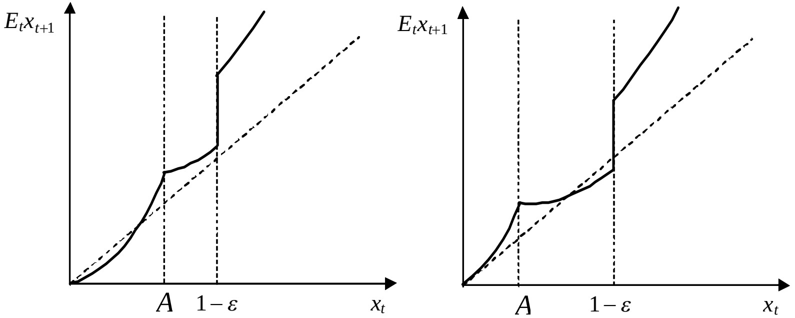
Existence of Bubbles:





Economic Growth with Bubbles

Existence of Bubbles:



Bubbles under Limited Arbitrage

Abreu and Brunnermeier (Econometrica, 2003), Bubbles and Crashes

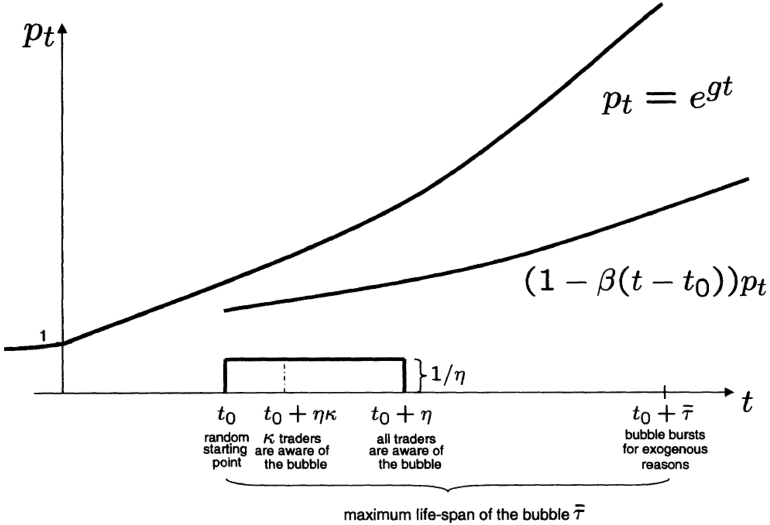
- Rational arbitrageurs interact with behavioral traders
- Lack of common knowledge: informed arbitrageurs don't know if other arbitrageurs are informed
- Arbitrageurs have difficulty coordinating their behavior
- Trade-off between two conflicting objectives:
  - make profits from riding the bubble
  - avoid losses by leaving the market in time
- Unique equilibrium of when the bubble will burst
- Small news events can trigger synchronization

Bubble Process

Assumptions on evolution of bubble and fundamental:

- Price starts at $t = 0$ with $p_0 = 1$
- During $t \in [0, t_0]$, price grows at rate g because of good fundamentals: $p_t = e^{gt}$
- t_0 is random, distributed exponentially over $[0, \infty)$ with c.d.f $\Phi(t_0) = 1 - e^{-\lambda t_0}$
- After t_0, fundamental grows at 'regular' rate $r < g$ discrepancy is 'bubble term' $\beta(t - t_0) = 1 - e^{-(g-r)(t-t_0)}$
- Stock price continues to grow at rate g until selling pressure of arbitrageurs sufficiently strong or until maximum bubble $\bar{\beta}$ reached at $\bar{\tau} \longrightarrow$ price drops by $\beta(\cdot)$

Bubble Process

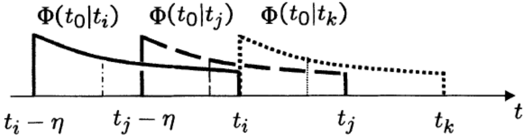


Arbitrageurs

Assumptions on information of arbitrageurs:

- Arbitrageurs become sequentially informed of bubble in random order over the interval $[t_0, t_0 + \eta]$ ... 'awareness window'
- Agent who receives signal at t_i knows that $t_0 \in [t_i - \eta, t_i]$
- Subjective distribution of t_0 is

$$\Phi\left(t_0 \middle| t_i\right) = \frac{e^{\lambda\eta} - e^{\lambda(t_i - t_0)}}{e^{\lambda\eta} - 1}$$



Arbitrageurs

Assumptions on behavior of arbitrageurs:

- Denote each arbitrageur's selling decision as $\sigma(t, t_i) \in \{0, 1\}$
- Each arbitrageur's selling time is $T(t_i) = \inf\{t | \sigma(t, t_i) > 0\}$
- Aggregate 'selling pressure' is $s(t, t_0) = \int_0^{\min\{t, t_0+\eta\}} \sigma(t, t_i)\, dt_i$
- Bursting time is $T^*(t_0) = \inf\{t | s(t, t_0) \geq \kappa \text{ or } t = t_0 + \bar{\tau}\}$
  where κ is the necessary aggregate selling pressure to pop bubble
  Note: $T^*(t_0)$ is strictly increasing
- Belief distribution of bursting time is $\Pi(t|t_i) = \int_{T^*(t_0) \leq t} d\Phi(t_0|t_i)$
- Transaction cost is constant in real terms: $c \cdot e^{rt}$
- Refinement of beliefs: if agent t_i sells, she rationally believes that all agents t_j, $j < i$ have already sold

$$\text{Implication:} \quad T^*(t_0) = \min\{T(t_0 + \eta\kappa), t_0 + \bar{\tau}\}$$

Arbitrageurs

Preemption:

- Arbitrageur t_i believes at her selling time $T(t_i)$ that at most a mass κ of arbitrageurs received the signal before her
- Otherwise, she knows that the bubble would already have burst before

Trigger-Strategy:

- Arbitrageur t_i will maintain her position $\sigma = 1$ for all $t \geq T(t_i)$

Expected payoff to selling at time t

$$\int_{t_i}^{t} e^{-rs} \underbrace{\left[1-\beta\left(s-T^{*-1}(s)\right)\right] p(s)}_{\text{fundamental price (bubble has burst)}} d\Pi(s|t_i) + e^{-rt} \underbrace{p(t)}_{\text{inflated price (no burst)}} \left[1-\Pi(t|t_i)\right] - c$$

Sell-out condition (first-order condition):

$$\text{sell if} \quad \underbrace{h(t|t_i)}_{\text{hazard rate}} > \frac{g-r}{\beta\left(t-T^{*-1}(t)\right)} \quad \text{where} \quad h(t|t_i) = \frac{\pi(t|t_i)}{1-\Pi(t|t_i)}$$

Exogenous Crashes

Condition for exogenous crashes:

$$\frac{\lambda}{1-e^{-\lambda\eta\kappa}} \leq \frac{g-r}{\bar{\beta}}$$

Unique trading equilibrium: sell at time $t_i + \tau^1$ where

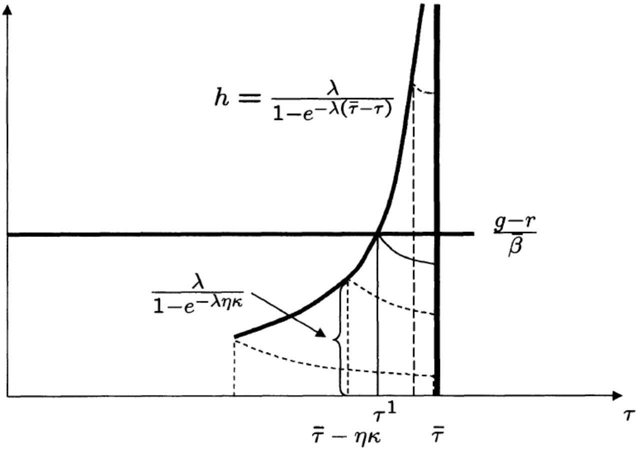
$$\tau^1 = \bar{\tau} - \frac{1}{\lambda}\ln\left(\frac{g-r}{g-r-\lambda\bar{\beta}}\right) < \bar{\tau}$$

BUT: Bubble bursts exogenously at $\bar{\beta}$ because $t_0 + \tau^1 + \eta\kappa > t_0 + \bar{\tau}$

Reason: subjective hazard rate of crash is sufficiently low:

$$h\left(t_i + \tau | t_i\right) = \frac{\pi\left(t_i + \tau | t_i\right)}{1 - \Pi\left(t_i + \tau | t_i\right)} = \frac{\lambda}{1-e^{-\lambda\eta\kappa}}$$

Equilibrium exit time for each arbitrageur:



Endogenous Crashes

Condition for endogenous crashes:

$$\frac{\lambda}{1 - e^{-\lambda\eta\kappa}} > \frac{g - r}{\bar{\beta}}$$

Selling at time $t_i + \tau^1$ with τ^1 as defined above is suboptimal, because everybody would know that bubble will burst at $t_0 + \tau^1 + \eta\kappa < t_0 + \bar{\tau}$

$\rightarrow$ sell before at $\tau^2 < \tau^1$, but this makes it optimal to sell even earlier

$\rightarrow$ selling time converges to $0 < \tau^* < \cdots < \tau^2 < \tau^1$

At time of crash: bubble is β^* defined by $\frac{\beta^* \lambda}{1 - e^{-\lambda\eta\kappa}} = g - r$

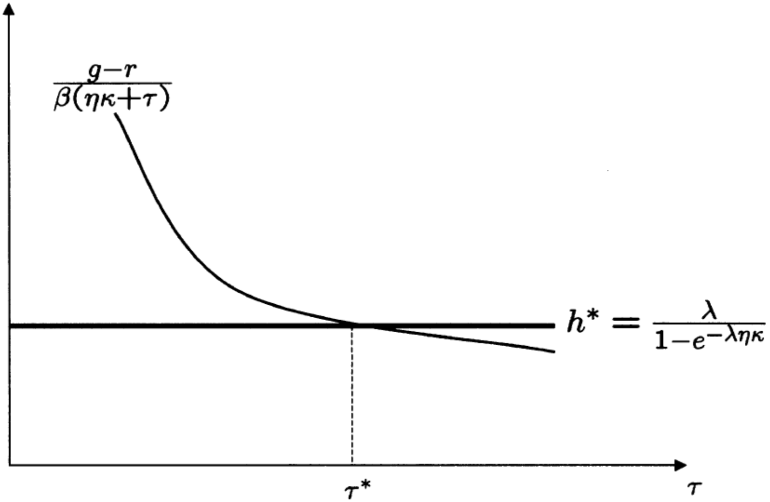
$\rightarrow$ this pins down an optimal selling time τ^*

$\rightarrow$ at time $t_0 + \tau^* + \eta\kappa$ a sufficient mass has sold to burst bubble

$\rightarrow$ hazard rate $h^* = \frac{\lambda}{1 - e^{-\lambda\eta\kappa}}$ constant

benefit/cost ratio $\frac{g - r}{\beta(\eta\kappa + \tau^*)}$ declining

Equilibrium exit time for each arbitrageur:



Synchronizing Events

Role of synchronizing events:

- Main problem for arbitrageurs: lack of common knowledge
  $\longrightarrow$ prevents coordination of attacks on the bubble
- Uninformative events (including price declines) can serve as a synchronizing event if agents believe it
  $\longrightarrow$ theory of 'overreaction' to news, of fads and of fashions
- Assume synchronizing signal with Poisson arrival rate of θ can be recognized only by informed arbitrageurs
- If coordinated attacks fail, bubble is strengthened

Responsive equilibrium:

- Equilibrium where all arbitrageurs believe that all others respond to event is unique (otherwise multiplicity of equilibria)
- All informed arbitrageurs sell out when synchronizing signal received
- If attack on bubble fails, they all re-enter the market
- In absence of signal they sell at equilibrium time τ^{**}

Farhi-Tirole (2010): Bubbly Liquidity

Main elements:

- Holmstrom-Tirole-type entrepreneurs in OLG
- financial frictions imply shortage in stores of value
  $\longrightarrow$ potential for development of bubbles
- bubbles provide valuable liquidity service for entrepreneurs
  $\longrightarrow$ potential that bubbles "crowd in" investment

Contribution (1)

Possibility of bubbles

- In traditional models of rational bubbles (e.g. Tirole, 1985): necessary condition for existence of bubble:

$$\text{interest rate } r \leq \text{growth rate } g$$

$=$ condition for dynamic inefficiency
$=$ condition for infinite NPV of rents

- Quest to sever the link between bubbles & inefficiency recipe: difference between private and social return to capital

$$\text{interest rate } r \leq \text{growth rate } g < \text{ social return } R$$

$\rightarrow$ new condition for possibility of bubbles

Contribution (2)

Real effects of bubbles

- In traditional models, bubbles are alternative store of value
  $\longrightarrow$ compete with real investment
  $\longrightarrow$ reduce investment (though increase efficiency)

- **Here: bubbles have a dual function**:
  - compete with investment as a store of value
  - provide liquidity to (future) entrepreneurs

  if entrepreneurs are net demanders of liquidity,
  they benefit from additional supply of liquidity
  $\longrightarrow$ investment increased

Model Setup

OLG of 3-period-lived entrepreneurs:

1. **young:** obtain endowment A and save it in stores of value:

$$A(1+r_t) = \underbrace{\hat{l}_t}_{\text{outside liquidity}} + \underbrace{\hat{b}_t}_{\text{bubbles}} + \underbrace{\hat{x}_t}_{\text{inside liquidity}}$$

   $\longrightarrow$ asset demand

2. **middle-aged:** used these funds for investment, where pledgeable return $\rho_0 <$ total return ρ_1

$$i_t = \frac{\rho_0}{1+r_{t+1}} \cdot i_t + A(1+r_t) \qquad \text{or} \qquad i_t = \frac{A(1+r_t)}{1-\frac{\rho_0}{1+r_{t+1}}}$$

   $\longrightarrow$ asset supply

3. **old:** realize return on investment $\rho_1 i_t$ and consume

Phase Diagram Analysis

Evolution of bubble: $b_{t+1} = (1 + r_{t+1})\, b_t$ by no-arbitrage condition

System can be described in two state variables (b_t, i_{t-1})

Conditions for steady state:

1. $i_t = i_{t-1}$
2. $b_{t+1} = (1 + r_{t+1})\, b_t$

$$\Rightarrow \quad \begin{cases} b^* = 0 & \text{or} \\ r^* = 0 \end{cases}$$

$\longrightarrow$ analyze dynamic system

Liquidity Provision vs. Competition Effect

Effects of outside liquidity:
Greater availability of stores of value raises interest rate r

1. Liquidity provision: higher return on wealth stored by the young
   $\longrightarrow$ "net worth effect" for middle-aged entrepreneurs
   $\longrightarrow$ tends to increase investment
2. Competition: more expensive to borrow for investment
   $\longrightarrow$ tends to reduce investment

In bubble-free steady state: $$i^* = \frac{A\left(1+r^*\right)}{1-\frac{\rho_0}{1+r^*}}$$

Liquidity effect dominates if $1+r^* > 2\rho_0$

Possibility of Bubbles

Standard condition:

- bubble is possible if $r^* < 0$ in no-bubble steady-state
- in terms of fundamental parameters:

$$\frac{1 - 2\rho_0}{1 - \rho_0} > \frac{I}{A}$$

(satisfied for low ρ_0, I and high A)

Effect of bubble:

- increase r, in steady-state to $r^{**} = 0$

(Note: bubbles possible under low interest rates,
but without bubble, interest rates are even lower!)

Macro Effects of Bubbles

Macro implications:

If firms are **net demanders of liquidity** ($l > 0$):

- higher interest rate due to bubble raises investment (net worth effect dominates)
- bubble and investment are complements

If firms are **net suppliers of liquidity** ($l < 0$):

- higher interest rate reduces investment (competition effect dominates)
- bubble and investment are substitutes

Extensions

Extensions:

- heterogeneity in pledgeability:
  firms with little pledgeability benefit more from bubble
  (b/c net worth effect relatively more important)

  note: there may be selection by productivity

- trade-off between pledgeability and value creation:
  bubble raises $r \longrightarrow$ value creation becomes relatively more important than pledgeability

- stochastic bubbles: trade at a "risk premium"
  because liquidity most valuable when they burst

  also: fundamental shocks to A may trigger burst of bubble