The Macroeconomics of Imperfect Capital Markets

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Lecture 3: Perfect Capital Markets

Review

Adrian, Colla and Shin: Which Financial Frictions?

- important distinction between bank finance and market finance
- during crisis: financial intermediation collapsed
  for large corporations, direct finance increased
- spreads on both types of finace rose sharply
- variations in bank credit supply driven by changes in leverage

Perfect Capital Markets

Classical Arrow-Debreu world:

- complete markets
- perfect rationality
- full information

Among the first important results:

- Fisher (1930): separation theorem
- Modigliani-Miller (1958): irrelevance of firms' financial policy

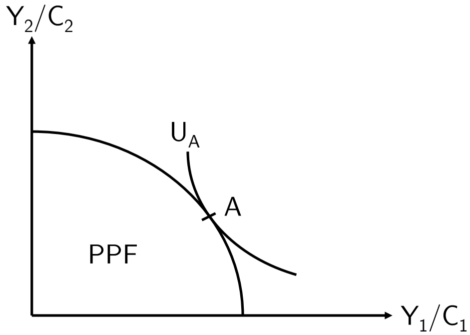
Fisher's Separation Theorem

Theorem (Fisher, 1930, Theory of Interest)

In perfect capital markets, a firm's financing/investment decision and its owner's saving/consumption decision can be analyzed separately.

1. *The firm's objective function is to maximize its market value, given its PPF.*
2. *The owner's decision is to maximize utility, given her budget constraint.*

Fisher's Separation Theorem



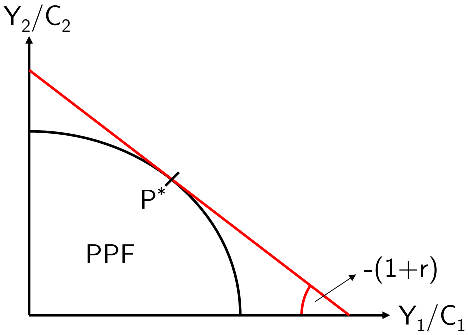
In financial autarky:

A = production = consumption

Note: entrepreneur's saving decision and

firm's investment decision integrated

Fisher's Separation Theorem



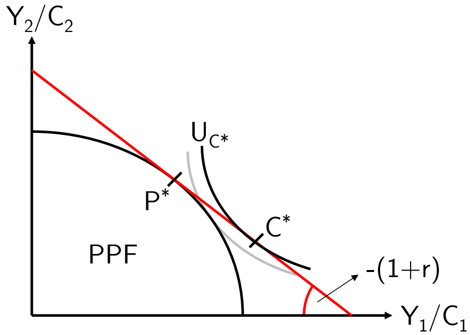
Under perfect capital markets: analyze problem in 2 steps:

$P^* =$ optimum level of production given PPF and interest rate $1 + r$

$\rightarrow$ firm value is maximized ("unanimity principle")

Note: production decision independent of firm's financing

Fisher's Separation Theorem



$C^* =$ optimum level of consumption given wealth and $1 + r$
$\rightarrow$ utility is maximized
Note: production decision independent of entrepreneur's preferences

Fisher's Separation Theorem

Analytic Version:

- Problem can be thought of across time $t = 1, 2$ with $p_2 = \frac{p_1}{1+r}$ or across states of nature $i = 1, 2$ with p_1, p_2 reflecting state prices (which depend on probabilities and risk aversion)
- Utility function $u(C_1, C_2)$
- Production possibilities frontier $F(Y_1, Y_2) = 0$

Autarky solution:

$$\max u(C_1, C_2) \text{ s.t. } F(C_1, C_2) = 0$$

Market solution given price system (p_1, p_2):

$$\begin{aligned} \max \quad & u(C_1, C_2) \text{ s.t. } p_1 C_1 + p_2 C_2 = \Pi \\ \max \quad & \Pi = p_1 Y_1 + p_2 Y_2 \text{ s.t. } F(Y_1, Y_2) = 0 \end{aligned}$$

A Few Definitions

Complete markets:
an economy in which there is an unconstrained market for every good
(note: definition of good includes time/state/location/etc.)

Dynamically complete market:
a system in which a self-financing trading strategy (involving re-trading) can be deviced for every good
(example: Arrow securities = 1 period ahead securities for every state of nature)

Effectively complete system of markets:
a system in which sufficient securities exist to achieve the same allocation that would be achieved under complete markets

Pricing Kernels

Modern asset pricing setup:

- representative agent who holds $\alpha = 1$ units of a risky security X with future payoff R_X
- solve optimization problem to determine equilibrium market price P_X

$$\max_{\alpha} u(C_1) + \beta E\left[u\left(C_2^{\omega}\right)\right] \quad \text{s.t.} \quad \begin{aligned} C_1 &= \bar{C} - \alpha P_X \\ C_2^{\omega} &= \bar{C} + \alpha R_X^{\omega} \end{aligned}$$

- optimality condition:

$$P_X = E\left[\frac{\beta u'\left(C_2^{\omega}\right)}{u'(C_1)} \cdot R_X^{\omega}\right] = E\left[M^{\omega} R_X^{\omega}\right]$$

where we define $M^{\omega} = \frac{\beta u'(C_2^{\omega})}{u'(C_1)}$, which is called

  - pricing kernel or
  - intertemporal marginal rate of substitution (IMRS) or
  - stochastic discount factor

Pricing Kernels

Interpretation of pricing kernel $M^{\omega} = \frac{\beta u'(C_2^{\omega})}{u'(C_1)}$**:**

- period 1 valuation of a unit payoff in state ω of period 2
- for risk-neutral agents, $M^{\omega} = \beta$
- for risk-averse agents, M^{ω} is high in low consumption states of period 2 and low in high consumption states

Bankruptcy and Economic Efficiency

Example (loosely based on Stiglitz, 1969):

Case 1: No bankruptcy

- Lenders and firms both have pricing kernel M^ω
- Firms have production technology $A^\omega F(K)$
- Lenders willing to lend at risk-free gross rate $R = \frac{1}{E[M^\omega]}$
- The objective of *both* the *firm* and the *social planner* is to solve

$$\begin{aligned} \max_K \quad & E\left\{M^\omega \cdot \left[A^\omega F(K) - RK\right]\right\} = \\ = \max_K \quad & E\left[M^\omega A^\omega\right] F(K) - E\left[M^\omega\right] RK = \\ = \max_K \quad & \bar{A} F(K) - K \end{aligned}$$

where we define $\bar{A} = E\left[M^\omega A^\omega\right]$ as investors' discounted valuation of the risky payoff A^ω.

Note: decentralized equilibrium is socially efficient

Bankruptcy and Economic Efficiency

Case 2: Possibility of bankruptcy

- Lenders and firms both have pricing kernel M^ω
- Lenders charge a risk-adjusted gross interest rate R_a
- Firms' objective under limited liability is to solve

$$\max_K E\left\{M^\omega \cdot \max\left[0, A^\omega F\left(K\right) - R_a K\right]\right\}$$

- The firm's repayment is

$$\text{where } R^\omega = \begin{cases} R_a K & \text{if } A^\omega F\left(K\right) \geq R_a K \\ A^\omega F\left(K\right) & \text{if } A^\omega F\left(K\right) < R_a K \end{cases}$$

- Lenders are willing to participate if R_a is such that

$$E\left[M^\omega R^\omega\right] = K$$

- Use the definition of R^ω to re-write firm's objective as

$$\begin{aligned} \max_K \quad & E\left\{M^\omega \cdot \left[A^\omega F\left(K\right) - R^\omega\right]\right\} = \\ = \quad & E\left[M^\omega A^\omega\right] F\left(K\right) - E\left[M^\omega R^\omega\right] = \bar{A} F\left(K\right) - K \end{aligned}$$

Bankruptcy and Economic Efficiency

Case 2: Alternative solution method:

- Solve for $R_a K$ from lenders' participation constraint:

$$K = E\left[M^\omega R^\omega\right] = \int_{nobkt} M^\omega R_a K dP^\omega + \int_{bkt} M^\omega A^\omega F\left(K\right) dP^\omega$$

$$\rightarrow \ R_a K = \frac{K - \int_{bkt} M^\omega A^\omega F\left(K\right) dP^\omega}{\int_{nobkt} M^\omega dP^\omega}$$

- Substitute for $R_a K$ in firms' maximization problem (in fourth line):

$$\begin{aligned}
\max_K \quad & E\left\{M^\omega \cdot \max\left[0, A^\omega F\left(K\right) - R_a K\right]\right\} = \\
= \ & \int_{nobkt} M^\omega \cdot \left[A^\omega F\left(K\right) - R_a K\right] dP^\omega = \\
= \ & \int_{nobkt} M^\omega A^\omega F\left(K\right) dP^\omega - \underbrace{R_a K} \int_{nobkt} M^\omega dP^\omega = \\
= \ & \int_{nobkt} M^\omega A^\omega F\left(K\right) dP^\omega - K + \int_{bkt} M^\omega A^\omega F\left(K\right) dP^\omega = \\
= \ & E\left[M^\omega A^\omega\right] F\left(K\right) - K = \bar{A} F\left(K\right) - K
\end{aligned}$$

Bankruptcy and Economic Efficiency

The solution with bankruptcy (and identical pricing kernel for firms and lenders) coincides with the no-bankruptcy case and the social planner case

Result: even with bankruptcy, firms' choice of K is socially efficient

Intuition:

- Interest rate adjusts upwards as bankruptcy risk rises
- In the decentralized equilibrium, R_a always conveys the correct price signal to the firm, i.e. it accurately reflects the social cost of funds

First Welfare Theorem

- Economy with n consumers, s states of nature
- Each agent i has endowment ω_i (s-vector)
- Agent i's consumption bundle $x_i = \left(x_i^1, ..., x_i^s\right)$
- Allocation $x = (x_1, ..., x_n)$ is feasible if $\sum x_i = \sum \omega_i$
- Utility function $u_i(x_i)$ is locally non-satiated
- price vector p

- Consumer demand function:
  $x_i(p) = \arg\max_{x_i} u(x_i) \quad \text{s.t.} \quad p(x_i - \omega_i) \leq 0$

- Definition of Walrasian equilibrium: pair (x, p) s.t.
  1. x is feasible: $\sum_i x_i = \sum_i \omega_i$
  2. x_i is optimal given p for all i: if $x_i' \succsim x_i$ then $px_i' > p\omega_i$

First Welfare Theorem

Theorem

If (x, p) is a Walrasian equilibrium, then x is Pareto efficient.

Proof.

- Suppose x' is feasible and weakly preferred by all agents (strictly preferred by agent j)
- By point 2 of definition of equilibrium, $px_i' \geq p\omega_i$ for all i and strictly for agent j
- Summing over all i, $p\sum_i x_i' > p\sum_i \omega_i$

$\rightarrow$ contradiction to feasibility $p\sum_i x_i' \leq p\sum_i \omega_i$ □

Intuition:

- since each agent optimizes (point 2 of definition), any allocation that makes agent j better off must be outside her budget set and infeasible

Characterization of Equilibrium and Efficiency

Proposition (Characterization of Market Equilibrium with Calculus)

If (x, p) is a market equilibrium with nonzero allocations, then $\exists$ $(\lambda_1, ...\lambda_n)$ s.t.

$$Du_i(x) = \lambda_i p$$

Note:

- the result follows from each agent's first-order conditions
- the λ_i's can be interpreted as marginal utilities of income

Equilibrium must satisfy

$$\frac{\frac{\partial u_i(x_i^*)}{\partial x_i^g}}{\frac{\partial u_i(x_i^*)}{\partial x_i^h}} = \frac{p_g}{p_h}$$

Characterization of Equilibrium and Efficiency

Proposition (Pareto Efficiency)

An allocation x^* is Pareto efficient if and only if x^* solves for $i = 1, ..., n$

$$\begin{aligned} \max_{x_i, x_j} u_i(x_i) \quad s.t. \quad \sum_{h=1}^{n} x_h^g &\leq \omega^g \ \forall g = 1, ..., k \\ u_j(x_j) &\geq u_j(x_j^*) \quad j \neq i \end{aligned}$$

Lagrange formulation:

$$\mathfrak{L} = u_i(x_i) - \sum_{g=1}^{k} q^g \left[\sum_{i=1}^{n} x_i^g - \omega^g \right] - \sum_{j \neq i} a_j \left[u_j(x_j^*) - u_j(x_j) \right]$$

FOCs: $\dfrac{\partial u_i(x_i^*)}{\partial x_i^g} - q^g = 0$ and $a_j \dfrac{\partial u_j(x_j^*)}{\partial x_j^g} - q^g = 0$ for $j \neq i$

$\rightarrow$ pick $p^* = q =$ shadow prices on resource constraint

$\rightarrow$ pick $a_i = 1/\lambda_i =$ welfare weight of agent i

Characterization of Equilibrium and Efficiency

Proposition (Welfare Maximization)

For any Pareto efficient allocation $x^* \gg 0$, there exist welfare weights $(a_1^*, ...a_n^*)$ such that x^* maximizes the welfare maximization problem

$$\max_x \sum_i a_i^* u_i (x_i) \quad s.t. \quad \sum_{i=1}^n x_i^g \leq \omega^g \forall g = 1, ..., k$$

The welfare weights a_i^* represent the inverse of the marginal utility of the agent, i.e. $a_i^* = 1/\lambda_i^*$.