The Macroeconomics of Imperfect Capital Markets

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Lecture 4: Incomplete Markets

Review

Perfect Financial Markets

- Fisherian separation:
  in perfect capital markets, investment and consumption decisions can be made independently

- Bankruptcy:
  in absence of frictions, bankruptcy risk does not distort investment decisions

Incomplete Markets

Classical Arrow-Debreu world:

- complete markets
- perfect rationality
- full information

Among the first important results:

- Fisher (1930): separation theorem
- Modigliani-Miller (1958): irrelevance of firms' financial policy

Hart (1975), Incomplete Markets

Hart (JET, 1975), On the Optimality of Equilibrium when the Market Structure is Incomplete

- important condition for welfare theorems: complete markets
- under incomplete markets:
    - equilibrium may not exist
    - equilibrium is generically inefficient
    - opening new markets may reduce welfare

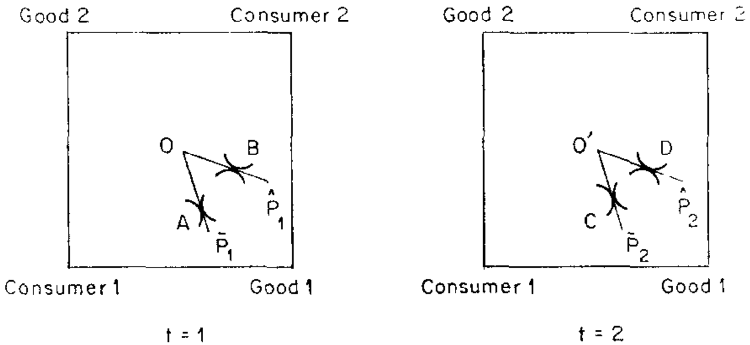
Example of Suboptimality

A simple example of suboptimality

- Two time periods $t = 1, 2$, no uncertainty
- two consumers $i = 1, 2$
- two goods $j = 1, 2$
- only spot markets, no intertemporal markets
- endowments $\omega_t^i \in R_+^2$ and consumption $x_t^i \in R_+^2$
- time-separable preferences $U^i = V^i\left(x_1^i\right) + \beta^i W^i\left(x_2^i\right)$
  $\rightarrow$ allows us to depict equilibrium in 2 Edgeworth boxes

- Pareto optimality: too strong of a benchmark:
  since there are no intertemporal markets, $MRS^1 \neq MRS^2$
- constrained Pareto optimality: optimal relative to the set of allocations that can be implemented with given markets

Example of Suboptimality



- assume each period has two possible equilibria – (A, B) and (C, D) $\rightarrow$ 4 possible combinations in total
- note: $B \succcurlyeq^1 A$ and $D \succcurlyeq^1 C$ and vice versa for agent 2
- if $\beta^1 < \beta^2$ then agent 1 cares more about the present than agent 2 $\rightarrow$ equilibrium (B, C) Pareto-dominates (A, D) for sufficiently low β^1

Benchmark Model

Environment:

- Three time periods $t = 1, 2, 3$
- Uncertainty captured by state of nature $s \in S$

Market structure:

- finite set of goods $j = 1, ...H$
- securities available at time 1 is $f = 1...F$ with payoffs a_2^f, a_3^f
- new securities available at time 2 are $f = F + 1, ...F + G$

Prices

- goods price system (p_1, p_2, p_3) where $p_t \in R_+^H$
- security price system (π_1, π_2) where $\pi_1 \in R_+^F$, $\pi_2 \in R_+^{F+G}$
- price system (p, π)

Benchmark Model

Consumers:

- Consumers $i = 1...I$
- endowment stream $\omega^i = \left(\omega_1^i, \omega_2^i, \omega_3^i\right)$
- consumption plan $x^i = \left(x_1^i, x_2^i, x_3^i\right)$
- security trading plan $z = \left(z_1^i, z_2^i\right)$
- all variables for $t \geq 2$ depend on state of nature $s \in S$
- utility function $U^i\left(x^i\right)$

Budget constraints:

$$\begin{aligned} p_1 x_1^i + \pi_1 z_1^i &\leq p_1 \omega_1^i \\ p_2(s)\, x_2^i(s) + \pi_2(s)\, z_2^i(s) &\leq p_2^i(s)\left[\omega_2^i(s) + z_1^i a_2\right] + \pi_2(s)\, z_1^i(s) \\ p_3(s)\, x_3^i(s) &\leq p_3^i(s)\left[\omega_3^i(s) + z_2^i a_3\right] \end{aligned}$$

feasible set $B^i\left(p, \pi, \omega^i\right)$

Benchmark Model

An **equilibrium** (under rational expectations) is an array (x, z) and a price system (p, z) such that:

- $\left(x^i, z^i\right) \in B^i\left(p, \pi, \omega^i\right)\ \forall i$ (feasibility)
- for each i, $U^i\left(x^i\right) \geq U^i\left(x'^i\right)$ for all $x'^i \in B^i\left(p, \pi, \omega^i\right)$ (optimality)
- $\sum_i x_t^i(s) \leq \sum_i \omega_t^i$ for all t, s (goods market clearing)
- $\sum_i z_t^i(s) \leq 0$ for all t, s (security market clearing)

A **Pareto optimum** is a consumption allocation x that is not Pareto dominated, i.e. $\nexists x'$ s.t. $U^i\left(x'^i\right) > U^i\left(x^i\right)\ \forall i$

Example of Non-Existence:

- Assume 2 consumers, 2 goods, 2 states with probability $\frac{1}{2}$
- consumption only at state $t = 2$ (like a 2 period economy)

$$\begin{aligned} U^1 &= E_s\left[\sqrt{8x_1^1(s)} + \sqrt{x_2^1(s)}\right] \\ U^2 &= E_s\left[\sqrt{x_1^2(s)} + \sqrt{8x_2^2(s)}\right] \end{aligned}$$

- endowments $\omega^1(1) = \omega^2(2) = \left(\frac{5}{2}, \frac{50}{21}\right)$ and $\omega^1(2) = \omega^2(1) = \left(\frac{13}{21}, \frac{1}{2}\right)$
- two securities with $a^1(s) = (1, 0)\ \forall s$ and $a^2(s) = (0, 1)\ \forall s$

Claim: equilibrium does not exist!

Example of Non-Existence

Sketch of proof:
observe monetary payoffs of securities $f = 1, 2$ are $p_1(s)$ and $p_2(s)$

1. consider case that $p(1)$ and $p(2)$ are linearly independent
   - then the two securities span the state space
   - equilibrium exhibits perfect risk-sharing
   - given symmetry, $\frac{p_1(1)}{p_2(1)} = \frac{p_1(2)}{p_2(2)} \longrightarrow$ linearly dependent!
2. consider case that $p(1)$ and $p(2)$ are linearly dependent
   - the two securities have the same span
   - no way to transfer wealth across states $s = 1, 2$
   - autarky equilibrium
   - then $p(1) = \left(\frac{2}{3}, \frac{1}{3}\right)$ and $p(2) = \left(\frac{1}{3}, \frac{2}{3}\right) \longrightarrow$ linearly independent!

Conclusion: since neither case is possible, no equilibrium exists!

Example of Non-Existence

Intuition:

- as long as prices are linearly independent, agents can insure
- as agents approach full insurance, prices $|p(1) - p(2)| \to 0$
- to obtain additional insurance, agents need to buy larger and larger asset positions with $\left|z_f^i\right| \to \infty$
- in the limit, the span of the state space collapses
  $\to$ but then insurance is no longer possible
- technically, budget correspondence $B(\cdot)$ is not upper-hemicontinuous

Note: existence could be restored if we impose a bound $\left|z_f^i\right| \leq L \ \forall f, i$, but this bound is arbitrary and has no economic foundations

Optimality of Equilibrium

Optimality:

Main difficulty: defining the appropriate benchmark for optimality

- if markets are incomplete, first-best generally cannot be obtained
- this paper: defines feasible allocations as those that can be achieved as competitive equilibria
- $E(\omega)$ ... set of competitive equilibria that result from endowments ω, i.e. $x \in E(\omega)$ iif $\exists\ (z, p, \pi)$ such that (x, z) is feasible, optimal and clears markets

Definitions of optimality:

- given ω, x is weakly optimality if $\nexists x' \in E(\omega)$ that Pareto-dominates x (but this is a weak concept, e.g. it holds whenever $E(\omega)$ unique)
- given ω, x is strongly optimality if $\nexists x' \in E(\omega')$ that Pareto-dominates x for some redistribution of endowments that satisfies $\sum_i \omega^i = \sum_i \omega'^i$

Optimality of Equilibrium

Two examples:

Example 1:

- two goods and two time periods
- multiple equilibria with linearly dependent/independent prices similar to our earlier example
  $\longrightarrow$ if equilibria exist and can be Pareto-ranked, one is constrained inefficient

Example 2:

- single good but three time periods
- trades in earlier periods affect marginal utility in later periods
- this distorts prices in spot markets in later periods
- if span of securities is altered $\longrightarrow$ reduced insurance possibilities
- but in competitive equilibrium, agents take prices as given

Optimality of Equilibrium

General Findings about Optimality:

Complete market structure:
existing securities can deliver any desired stream of payoffs
$\longrightarrow$ any equilibrium will be Pareto optimal

Complete market structure up to date $T-1$ *and only one good:*
$\longrightarrow$ any equilibrium will be strongly optimal

Note: these are the most general possible conditions for optimality

Opening New Markets

Consequences of Opening New Markets:

- naive presumption: opening new markets is always good
- BUT: if the end-result is not complete markets,
  opening markets may reduce welfare

Example in paper:
opening new markets reduces security span and destroys insurance opportunities
$\longrightarrow$ **all** consumers are worse off

More generally:
opening new markets affects relative prices and leads to income effects
$\longrightarrow$ these may go in adverse directions and make equilibrium less efficient