ECON747 – Problem Set 1
Due date: March 12th, 2013, at the beginning of class
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I) Bankruptcy Risk and Interest Rates: Consider a risk-neutral firm with a neoclassical production function $Af(k) = Ak^\alpha$. The productivity parameter A is a random variable with distribution $G(A)$ and support $[0, \infty)$. Capital k is used up in the process of production. The firm has a given level of equity e and borrows an amount d of standard debt to reach the desired level of capital $k = e + d$. Banks are risk-neutral, have perfect information and can dispose of a large endowment of funds. Furthermore, they have access to an alternative storage technology that yields a safe gross return of $1 + r$. (Note: You might not obtain explicit solutions to some of the problems. In that case, describe the functional forms you obtain as necessary to answer the questions.)

1. For any given level of e and any fixed gross interest rate R charged by the bank, (i) how much does the firm borrow and invest? (ii) Under what conditions will there be bankruptcy risk, and (iii) what would be the critical value of A below which the firm goes bankrupt? (Note: this firm's behavior is called interest rate-taking.)

2. Assume that banks are competitive and can costlessly extract all resources Ak^α in case of bankruptcy. Taking firm behavior as given, (i) express the interest rate $R(e, d)$ that banks charge to the firm as a function of their equity and debt position? (ii) Taking e as given, over what range of d is $R(\cdot)$ flat, monotonically increasing in d, and for what d does $R(\cdot)$ diverge to infinity? (iii) Interpret economically and (iv) depict graphically.

3. One day, the firm has a long conversation with its bank and realizes that the bank is not offering a fixed interest rate R, but in fact a menu $R(e, d)$ as you derived in question 2., where d is a choice variable of the firm. The firm immediately re-optimizes the problem given in question 1., internalizing the functional form of $R(\cdot)$ that you derived. (i) How much does the firm want to borrow and invest? (ii) In case your answer is different from question 1., show in which direction it differs and interpret economically why.

4. How much would a social planner invest in the given firm? Compare your answer to your results in questions 1. and 3. and interpret any differences/similarities. In particular, does bankruptcy risk distort the firm's investment decisions?

II) Costly Bankruptcy: Assume that we introduce a state verification cost c in case of bankruptcy, i.e. the firm's liquidation value is $Ak^\alpha - c$. (i) Rederive the new functional form of $R(\cdot)$. (ii) How much does the firm borrow and invest compared to the case without bankruptcy cost? Does bankruptcy risk distort the firm's investment decisions compared to the first-best allocation? (Heuristic answers sufficient.)