ECON747 – Problem Set 2
Due date: March 12th, 2013, at the beginning of class
Anton Korinek – `http://econ747.korinek.com`

Local Currency Debt as Insurance: Consider a small open economy in which there are two time periods, 0 and 1, and two goods, tradable and non-tradable goods. The tradable good is the numeraire, and the relative price of the non-tradable good p_N is a measure of the real exchange rate in the economy. In period 1 a state of nature $\omega \in \Omega$ is realized.

There are two sets of agents, international lenders and a representative domestic agent. International lenders are large in comparison to the small open economy, risk-neutral, and have a large amount of outside wealth in period 0 which they can carry from period 0 to period 1 using a storage technology with gross return 1. Their utility function is $V = E[W]$ where W is their tradable wealth in period 1. They have no use for non-tradable goods.

The domestic agent has no initial wealth in period 0, but he trades can trade two securities with international lenders in that period: one repays a unit of the tradable good in period 1 and represents foreign currency debt, the other repays a unit of the non-tradable good in period 1 and represents local currency debt. Note that foreign investors can trade bonds denominated in non-tradable goods and convert the proceedings into tradable goods at the prevailing exchange rate p_N^ω. The representative domestic agent values period 1 consumption according to the utility function $u(C_T, C_N)$, which is twice continuously differentiable and quasiconcave, and satisfies the Inada conditions.

In period 1, the domestic agent receives a stochastic endowment $A_T^\omega \in [A^{\min}, A^{\max}] \subset \Re^+$ of the tradable good that depends on the state of nature $\omega \in \Omega$ with expected value $E[A_T^\omega] = \bar{A}_T$, and a fixed endowment $\bar{A}_N = 1$ of the non-tradable good.

Foreign investors are willing to pay $E[p_N^\omega]$ tradable goods in period 0 for the promise of receiving one unit of local currency debt that delivers p_N^ω units of tradable goods in period 1. Denoting domestic holdings of local and foreign currency debt by L and F, the domestic budget constraint in period 0 and tradable income in period 1 are

$$F + LE[p_N^\omega] = 0 \qquad \text{and} \qquad Y_T^\omega = A_T^\omega - F - Lp_N^\omega$$

1. Write down the domestic agent's period-1 budget constraint for state ω and set up the Lagrangian for his optimization problem.

2. Compute the first order conditions (FOCs) of the representative agent's optimization problem. State the market clearing conditions for tradable and non-tradable goods, and using these together with the FOCs show that $p_N^\omega = p_N(C_T^\omega)$. Assuming tradables and nontradables are complements, show that $p_N'(Y_T^\omega) > 0$.

3. Define what constitutes an equlibrium in the described small open economy.

4. Solve for the equilibrium. Does it exist?

5. Provide some intuition for your answer to the previous question.

6. Describe the equlibrium in this small open economy if an upper bound for local currency bonds $L \leq \bar{L}$ is introduced. Does it exist? (Hint: How does the optimality condition for L change when $L \leq \bar{L}$?)